SOLUTIONS TO PROBABILITY QUESTION FROM LAST TIME

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Exercise 1. The letters of the alphabet are written on 26 cards. Two cards are chosen at random. What is the probability that at least one of them is a consonant?

There are different ways one can solve this; I'll write 3 possible solutions. Compare with what you've done. On an exam, you'd have to write only one correct solution, of course.

Solution 1.

Let
$$E = \text{event that at least one is a consonant.}$$
Let's compute
$$E^c = \text{event that neither is a consonant}$$
instead and then use
$$P(E) = 1 - P(E^c).$$
Let A be event that the first card is a consonant and B be event that the second card is a consonant. The event E^c is the event that the first card is not a consonant and the second card is not a consonant, so
$$E^c = A^c \cap B^c.$$
Note that A^c and B^c are dependent, and $P(A^c) = \frac{5}{26}$ and $P(B^c \mid A^c) = \frac{4}{25}$. Thus
$$P(A^c \cap B^c) = \frac{5}{26} \times \frac{4}{25} = \frac{2}{65}.$$
Thus
$$P(E) = 1 - \frac{2}{65} = \frac{63}{65}.$$
Note 1. We were after the probability of $E = A \cup B$. Instead, we computed the probability of $E^c = (A \cup B)^c$ and what we really did is use DeMorgan's law to write $(A \cup B)^c = A^c \cap B^c$.

Solution 2.

Let
$$E = \text{event that at least one is a consonant.}$$
Let A be event that the first card is a consonant and B be event that the second card is a consonant. , so
$$E = A \cup B.$$
Now A and B are not independent, so recall that
$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Now the unconditional probability of A or B is $P(A) = P(B) = \frac{21}{26}$. Also, we can compute

$$P(A \cap B) = P(A)P(A \mid B) = \frac{21}{26} \times \frac{20}{25}.$$

Therefore,

$$P(E) = \frac{21}{26} + \frac{21}{26} - \frac{21}{26} \times \frac{20}{25} = \frac{42 \cdot 25 - 21 \cdot 20}{26 \cdot 25} = \frac{63}{65}.$$

(This was not hard at all to simplify)

Solution 3.

Let

$$E = \text{event that at least one is a consonant.}$$

Let's compute

$$E^c = \text{event that neither is a consonant}$$

instead and then use

$$P(E) = 1 - P(E^c).$$

Now let's compute $P(E^c)$ in a different way. E^c is the event that neither card is a consonant, or equivalently, that both cards are vowels. There are $\binom{26}{2}$ ways to choose 2 cards out of 26, and there are $\binom{5}{2}$ ways to choose 2 vowels out of 5. Therefore,

$$P(E^c) = \frac{\binom{5}{2}}{\binom{26}{2}} = \frac{\frac{5!}{2!\ 3!}}{\frac{26!}{2!\ 24!}} = \frac{5!\ 2!\ 24!}{2!\ 3!\ 26!} = \frac{2}{65}.$$

(This was not hard at all to simplify)

Therefore,

$$P(E) = 1 - \frac{2}{65} = \frac{63}{65}.$$